VALIDATION OF AN ALTERNATIVE REFORCING DETAIL FOR THE DAPPED ENDS OF PESTRESSED DOUBLE TEES

A. Nanni and P. C. Huang

Center for Infrastructure Engineering Studies
University of Missouri-Rolla
224 Engineering Research Laboratory
Rolla, MO 65409-0710
Tel: (573) 341-4497
Fax: (573) 341-6215
e-mail: nanni@umr.edu

ABSTRACT

Prestressed concrete (PC) dapped-end beams have been used in buildings and parking structures as they provide an efficient and economical construction system. The reentrant corner of a dapped-end beam develops a severe stress concentration, which makes it the weakest point of the connection. If suitable reinforcement is not provided at this location, diagonal tension cracks may propagate rapidly and failure may occur with little or no warning. Reinforcing schemes and associated methods of design, which combine simplicity of application with economy of fabrication and provide the margin of safety required by present building codes, have been developed. This paper describes the experimental validation of an alternative reinforcing detail for the dapped-ends of prestressed double tees, which satisfies the requirement of the design method contained in the PCI Design Handbook.
INTRODUCTION

In recent decades, precast prestressed concrete (PC) structures have become more and more prevalent in the construction industry. The use of PC has been shown to be technically advantageous, economically competitive, and esthetically superior because of the reduction of cross-sectional dimension and consequent weight savings, enlargement of span length, cracking and deflection control, and larger shear force resistance. The use of precast concrete can improve the quality of the final product, decrease construction time and assist the progress of construction in adverse weather conditions. Unlike a cast-in-place reinforced concrete (RC) structure that is by its nature monolithic and continuous, a precast concrete structure is composed of individual prefabricated members that are connected by different types of joints. The type of connection used determines the behavior of a precast structure under load.

The design of dapped-end connections is one of the important considerations in a precast PC structure even though its analysis is complex. The shape of the dapped-end develops a severe stress concentration at the reentrant corner such that the conventional flexural theory is only partially applicable. In addition to the calculated forces from external loads, dapped-ends are also sensitive to horizontal tension forces arising from restraint of shrinkage or creep shortening of the member. Therefore, appropriate reinforcing schemes have to be investigated.

Limited research was conducted on dapped-end beams prior to 1969. In the 1970’s, a comprehensive research was carried out at the University of Washington that produced practical design criteria. A list of annotated publications on dapped-end members is shown in Appendix I. The objective of this study was to validate the use of the pattern of reinforcement shown in Fig. 2, to satisfy the reinforcement requirements of the design provisions for dapped-end members contained in the PCI Design Handbook, Fifth Edition.
CHARACTERISTICS OF DAPPED-END BEAMS

In this experimental program, two double tee PC beams with dapped-ends, SP I and SP II, were used. The beams were designed to carry a service load of 50 psf over a 60 ft long span, which corresponded to an ultimate shear demand $V_u$ of 24 kips per stem at the support. Each beam was prestressed by seven steel tendons in both stems (see Fig. 1). The properties of concrete, prestressed strands, and the dimensions of the beams as provided by the manufacturer are presented in Tables 1 and 2 and Fig. 1, respectively.

The mild steel reinforcement in the dapped-ends was incorporated at the time of the double tee construction. Details and amount were based on the design method presented in the PCI Design Handbook\(^3\). Two sizes of mild steel rebars were used: No. 6 bent bars were used for shear reinforcement (i.e., $A_{sh}$ and $A_v$ as defined in Fig. 3); and No. 7 bars were used for flexural and axial tension reinforcement (i.e., $A_s$ as defined in Fig. 3). Bars were fillet welded to a two-piece steel bearing plate, with dimensions of $5\frac{3}{8} \times 3\frac{3}{8} \times 6$ in. The horizontal reinforcement was anchored in the extended end in order to preclude bond failure. All rebars used were deformed and conformed to ASTM specification A706. The properties of mild steel as provided by the manufacture are listed in Table 3. The dimensions and reinforcement details are shown in Fig. 2.

The 1999 PCI Provisions\(^4\) propose five potential failure modes which should be investigated separately as numbered and shown in Fig. 3. Design of connections which are recessed or dapped into the end of the member greater than 0.2 times the height of the member ($H$ in Fig. 3), requires the investigation of all potential failure modes\(^2,4\). This design method is appropriate for cases where shear span-to-depth ratio ($a/d$ in Fig. 3) is not more than 1.0\(^3\). The calculations for the capacity of the dapped-end, based on the given mild steel reinforcement, are reported in Appendix II. From these calculations it is apparent that the
controlling case is that of a failure induced by diagonal cracking at the reentrant corner, for
which the stem capacity provided by the steel is $V_n = 52.8$ kips.

Since the stem shear demand is $V_u = 24$ kips, which is less than the factored strength
$\Phi V_n = 44.9$ kips, the dapped-end reinforcement as provided is considered satisfactory.

TEST SETUP AND PROCEDURE

The testing equipment used consisted of two 200-kip hydraulic cylinders and a
hydraulic pump for applying the load, linearly variable differential transformers (LVDT’s)
for measuring deflections, inclinometers for measuring rotations, extensometers for
measuring the change in width of cracks, pressure transducers and load cells for measuring
the applied load, strain gages for measuring strains, and a data acquisition system for
recording and storing data.

All tests used two hydraulic jacks placed under the nibs to push up the specimen
restrained by a steel spreader beam sitting on the specimen flange and anchored to the floor
by two steel bars (see Figs. 4 and 5). Bearing pads were placed on the jacks and plywood
sheets were placed under the steel spreader beam at loading points in order to protect the
concrete from localized damage. The first specimen, SP I, was tested with a shear span of 8
ft. and the second, SP II, was tested with the shear span of 5 ft. (see Figs. 6 and 7).

The test began with installing jacks and instrumentation. Once the instruments were
connected to the data acquisition system and the hydraulic cylinders were connected to the
pump, a preliminary load cycle was run with a relatively small load (less than 5 kips) to
insure that the equipment was functioning properly. The load test involved applying several
load cycles. Each load cycle consisted of a minimum of four approximately equal
incremental load steps followed by at least two steps for unloading. Each load step was
maintained for at least 1 minute and, during this time, deflection was monitored for stability.
TEST RESULTS AND DISCUSSIONS

Test results including first crack shear load, failure load, and failure mode are summarized in Table 4 and compared to predicted capacities.

Crack patterns and failure mode of specimen SP I

As the load increased to 28.22 kips per stem, the first crack began to form and extend at 45° angle at the reentrant corner. One crack formed near the support bearing plate, about 3.5 in. from the end face, and began to extend upwards. Flexural cracks, about 4 ft from the beam end, propagated and traveled upwards from the bottom of both stems. As the load was further increased, cracks in the nib began to turn and propagate at 45° towards the web-flange junction. Flexural cracks extended further towards the top spreader steel beam gradually.

At about 40 kips of applied load, cracking became extensive, especially those flexural cracks extending towards the spreader beam. As failure neared, the top face of the specimen flange, about 8 ft from the extended end, began to bulge and spall. At the maximum load of 45.68 kips per stem, the specimen failed due to shear-flexure failure rather than shear at the reentrant corner. The cracks in the nib and at reentrant corner did not extend further at maximum load and the reinforcement for the flexure and axial tension, $A_{st}$, appeared not to have yielded. The appearance of specimen SP I after failure is shown in Fig. 8 and the crack propagation is given in Fig. 9.

Crack patterns and failure mode of specimen SP II

For the test of specimen SP II, the loading span was reduced from 8 to 5 ft. in order to force a shear failure at reentrant corner. At 29.10 kips of applied load, the first crack at the reentrant corner began to form and extend upwards. One crack formed on the nib of the west stem, about 6 in. above the bearing plate, and began to extend upwards. Flexural cracks,
about 4 ft from the end face, propagated from the bottom of both stems and began to travel upwards to the top spreader steel beam.

As the load kept increasing, the first crack at the reentrant corner continued to extend. Inclined cracks in the nib extended from the bearing plate. Flexural cracks were more extensive and some cracks were found under the flange at the steel spreader beam. At 40 kips of applied load, inclined cracks from the full-depth portion reached the flexural and axial tension reinforcement $A_s$. Reentrant corner cracks extended upwards at a $45^\circ$ and began to link up with cracks on the nib.

As failure neared, about 50 kips per stem, the reentrant corner cracks reached the web-flange junction. Cracks extending from the full-depth portion joined the reentrant corner cracks and traveled upwards to the web-flange junction. As a maximum load of 57.40 kips per stem was reached, specimen SP II reached shear-flexure failure. The appearance of specimen SP II after failure is shown in Fig. 10 and the crack propagation is given in Fig. 11.

**OBSERVATIONS AND DISCUSSION**

Up to the formation of the first crack, each beam behaved elastically as shown in the traces of the loading-unloading cycles. In both cases, the first crack appeared at the reentrant corner. This was detected by visual observation of the concrete surface and by the reading of the $45^\circ$ strain gages/extensometers mounted at the reentrant corner.

In Fig. 12, the strain reading of gage 22 at the reentrant corner of specimen SP I shows a sudden increment at about 28 kips, considered the load corresponding to the formation of the first crack. As the load was further increased, other cracks formed and propagated from the bearing plate where the load was applied. Due to this, the strain readings of gages 21, 22, and 23 decreased to the point that the tip of the cracks monitored by
gages 21 and 23 closed (see negative strain readings in the figure). The shear cracks at the
reentrant corners remained narrow even at ultimate load.

In Fig. 13, the crack width measured by two extensometers at the reentrant corner on
the stems of specimen SP II shows a sudden jump when the applied load reached about 29
kips, the first crack load. Even at load levels close to ultimate, crack width remained
relatively small and close to the value specified by code provisions for interior exposure
G corresponding to a crack width of 0.016 in.

Figs. 14 and 15 show the envelope diagrams of load vs. net deflection per stem of
specimens SP I and SP II, respectively. The analytical lines plotted in the figures for
reference were based on the flexural stiffness of the uncracked specimen without accounting
for shear deformation. There is a good match between experimental and analytical curves up
to a load of approximately 20 kip.

In Fig. 14, the experimental envelope is interrupted at around 40 kips, even though the
beam failed in shear-flexure at a load of 47.5 kips, since the deflection gages were removed
to avoid damage. In the case of specimen SP II (Fig 15), significant deviation from linearity
occurred at a load close to 50 kips, even though the first crack at the reentrant corner occurred
at a load of 29 kips.

As indicated in Table 4, because both tests showed a shear-flexure failure, they did
not fully demonstrate the true ultimate capacity of the dapped-end. Specimen SP II showed
that the stem nominal shear capacity based on use of $f_y = 60$ ksi for reinforcing bars ($V_n =
52.8$ kips) could be attained prior to failure in the full-depth section of the beam. Further
reduction of the loading span (i.e., less than 5 ft) was considered not representative of the
behavior of a member designed to carry uniform distributed loads (udl). In this specimen, the
internal maximum moment and corresponding shear at the cross-section 5 ft away from the
support were 109 and 120% of the values corresponding to a udl that would produce the same
support reaction. This is to say that the test conditions resulting from the chosen geometry and set up were reasonably close to the field conditions in terms of absolute and relative internal forces generated at the critical full-depth cross-section.

CONCLUSIONS

The reentrant corner is a critical zone of any dapped-end PC beam. This project intended to provide a validation of the use of the alternative pattern of reinforcement shown in Fig. 2, to satisfy the requirement of dapped-end design according to PCI provisions. Based on the findings of the study, the following conclusions can be drawn:

- Both specimens failed in shear flexure in the full depth portion of the member before failure of the dapped end. However, in specimen SP II the shear in the dapped end at failure was 109% of the nominal strength of the dapped-end, calculated according to the PCI Design Handbook\(^4\) using the specified yield strength of 60ksi for the reinforcing bars.
- The shear demand of double Tee PC members up to 60 ft in length and carrying a live load of 50 psf is considerably lower than the capacity of the dapped-end having the reinforcement detail as tested.
- The reinforcement detail consisting of two No. 6 bent bars for shear reinforcement (A\(_{sb}\), and A\(_{sv}\)) and one No. 7 bar for flexural and axial tension reinforcement (A\(_s\)) was proven suitable.
- Anchorage of the steel reinforcement is critical and the present detail consisting of bars fillet welded to a two-piece end plate appears to perform satisfactorily.

CONVERSION FACTORS
ACKNOWLEDGEMENT

The authors acknowledge the contribution of Coreslab Structures Inc. (Oklahoma City, OK) in fabricating the beams, and the National Science Foundation (NSF) Industry / University Cooperative Research Center based at the University of Missouri Rolla in providing funding for the project.

The authors highly appreciated comments and suggestions from Dr. Alan H. Mattock, the pioneer researcher on the dapped end design.

REFERENCES


## APPENDIX I – Summary of Publications on Dapped-end Design

<table>
<thead>
<tr>
<th>Year/ Author</th>
<th>Thrust</th>
<th>Conclusions</th>
</tr>
</thead>
</table>
| 1969 Reynold¹      | Developed suitable reinforcement details evolving in a design procedure for dapped-end members. | 1. Diagonal stirrups provide suitable reinforcement.  
2. Joints can be designed based on equilibrium.  
3. Horizontal stirrups should be included to act against axial tension.  
4. Tensile reinforcement should be extended to the end of the beam to offer anchorage for stirrups. |
| 1970 Sargious and Tadros⁵ | Used finite element analysis to determine the behavior and strength of dapped-end beams. | 1. Several arrangements of prestressed cable profiles were proposed.  
2. No experimental validation. |
| 1973 Werner and Dilger⁶ | Determined first cracking shear at reentrant corner using FEM. Determined concrete contribution to cracking shear. | 1. Cracking shear at reentrant corner agreement with FEM using concrete tensile strength $6 \sqrt{f'_c} ; 4 \sqrt{f'_c}$ for practical design.  
2. Cracking load can be taken as contribution of concrete.  
3. Shear strength is the summation of concrete, shear reinforcement, and prestressing.  
4. Vertical and inclined shear reinforcement seems to be equally efficient in resisting shear. |
| 1975 Hamoudi, Phang, and Bierweiler⁷ | Developed the mechanics of diagonal shear cracks. | 1. Shear strength of prestressed dapped-ends can be predicted based on elastic analysis.  
2. Shear cracking load for beams with post-tensioned bars is equal to failure load.  
3. Beams with low values of reinforcement and high prestress failed in flexure, low prestressed beam failed by concrete rupture.  
4. With high strength steel reinforcement, shear cracking did not occur during working loads.  
5. Ultimate shear strength increased with an increase in prestress and a/d ratio. |
| 1979 Mattock and Chan² | Applied corbel design concepts to dapped-end beams. Determined concrete capacity and length of shear span “a”. | 1. The reduced depth of dapped-end may be designed as a corbel if “a” is measured to the center gravity of the hanger reinforcement.  
2. Closed stirrups $A_{sh}$ should be provided close to the end face to resist the vertical component of the inclined compression in the nib.  
3. The full-depth part of the beam should be designed to satisfy moment and force equilibrium.  
4. The nib reinforcement should be provided with a positive anchorage close to the end.  
5. The horizontal stirrups $A_h$ should be anchored near the end face.  
6. Concrete contribution should be ignored. |
| 1981 Khan⁸         | Verified previous design proposals for beams with a/d ratio ≤ 1.0, utilizing horizontal stirrups only in the nib. Verified beams having a/d ≥ 1, utilizing combined horizontal and vertical stirrups in the nib. | 1. Results obtained showed the validity of Mattock and Chan’s recommendation for beams with a/d ≤ 1.  
2. A $1.0 \leq a/d \leq 1.5$ dapped-end can be designed as a deep beam, using a combination of horizontal and vertical stirrups.  
3. The behavior of dapped-ends is in agreement with the assumption of a “truss-like” behavior. |
<p>| 1983 Liem⁹         | Studied maximum shear strength of a dapped-end or corbel with inclined | 1. Ultimate strength of a dapped-end with 45°inclined reinforcement should have twice the strength of a dapped-end with horizontal or vertical reinforcement. |</p>
<table>
<thead>
<tr>
<th>Year</th>
<th>Author(s)</th>
<th>Study Details</th>
<th>Highlights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>Chung</td>
<td>Used two a/d ratios and compared results to previous studies.</td>
<td>1. Mattock and Chan’s design leads to satisfactory behavior from strength and serviceability viewpoints in the case of h/H = 0.5; the hanger reinforcement carries the total shear. 2. Anchorage must be provided for both nib and beam flexural reinforcement at the end faces. 3. Horizontal stirrups alone are only satisfactory for nibs with a/d ≤ 1.0. 4. For a/d ≥ 1.0, the proposed ACI-426 design results in satisfactory behavior of dapped-end beam nibs.</td>
</tr>
<tr>
<td>1986</td>
<td>Ajina</td>
<td>Investigated cracking and shear capacity of connections with different patterns of shear reinforcement. Investigated the contribution of steel fibers in shallow end depths.</td>
<td>1. 1.2 % steel fibers content can be considered as a good substitute for vertical stirrups A_v. 2. Only h/H ≥ 0.5 should be allowed in dapped-end beams when steel fibers are not used. 3. Close stirrups should be used in all occasions.</td>
</tr>
<tr>
<td>1986</td>
<td>Mattock and Theryo</td>
<td>Investigated the behavior of a dapped-end at ultimate modeled using an analogous truss.</td>
<td>1. The behavior of a dapped-end can be modeled using an analogous truss. 2. Prestressing contribution can be included if 50 % of the strands pass through the nib and the hanger reinforcement is inclined at between 20 and 45 degrees to the vertical. 3. Vertical and inclined hanger reinforcement is equally efficient in resisting shear. Inclined hanger reinforcement is more effective in controlling cracking at service. 4. Provide a minimum 1.0 in. bottom concrete cover to hanger reinforcement instead of 0.75 in.</td>
</tr>
<tr>
<td>1988</td>
<td>Barton</td>
<td>Detailed dapped-end beams by using strut-and-tie design.</td>
<td>1. Ultimate shear capacity of strut-and-tie model is in the range of PCI’s. 2. Distribution of internal forces changed when load exceeded design capacity due the test method. 3. Anchorage requirements based upon the strut-and-tie model are conservative. Proper anchorage of the horizontal reinforcement is important.</td>
</tr>
<tr>
<td>1989</td>
<td>So</td>
<td>Studied different reinforcement details for thin-stemmed PC members using strut-and-tie model.</td>
<td>1. The strut-and-tie models were capable of estimating the failure load. 2. The use of two layers of symmetrically placed welded fabric notably improved ductility. 3. An inclined dapped-end is more efficient than a rectangular dapped-end.</td>
</tr>
<tr>
<td>1990</td>
<td>Mader</td>
<td>Studied and compared two design methods to PCI and other methods to determine how prestressing forces affect the load path in a beam.</td>
<td>1. All design methods resulted in beam ends that carried loads 15~20% higher than predicted except for the PCI method. 2. Standard hook details in the horizontal reinforcement within the dap-end provided adequate anchorage so that welding could be avoided.</td>
</tr>
<tr>
<td>2000</td>
<td>Huang</td>
<td>Validated PCI dapped-end design with full-scale tests. Proposed alternative strengthening method by using FRP composites.</td>
<td>1. Specimens with steel reinforcement failed in shear-flexure rather than shear failure at the reentrant corner. 2. Strengthening of the dapped-end using externally bonded FRP laminates with the option of U-anchor may allow flexibility in fabrication of PC members. 3. Proposed analytical expressions to evaluate the shear capacity of FRP strengthened dapped-ends are satisfactory and conservative.</td>
</tr>
</tbody>
</table>
APPENDIX II: REINFORCEMENT IN THE EXTENDED END

Flexural and axial tension

\[ A_s = \frac{1}{\phi f_y} \left[ V_u \left( \frac{a}{d} \right) + N_u \left( \frac{h}{d} \right) \right] \quad \text{(PCI Eq. 4.6.3)} \]

where

\[ \phi = 0.85 \]

a = shear span, in., measured from load to center of \( A_{sh} \)

h = depth of the member above the dap, in.

d = distance from top to center of the reinforcement \( A_s \), in.

\( f_y \) = yield strength of the flexural reinforcement, psi

From Fig. 2,

\[ a = 4\frac{1}{8} \text{ in.} + 1\frac{1}{2} \text{ in.} + \frac{1}{2} \times 6\frac{6}{8} = 6 \text{ in.} \]

\[ d = 12\frac{3}{8} \text{ in.} + 3\frac{1}{8} \text{ in.} - \frac{1}{2} \times 7\frac{7}{8} = 15.06 \text{ in.} \]

\[ \frac{a}{d} = \frac{6}{15.06} = 0.4 \text{ (since less than 1.0, OK)} \]

\[ h = 12\frac{3}{8} + 3\frac{1}{8} + \frac{3}{8} = 15.875 \text{ in.} \]

\( f_y = 60 \text{ ksi} \)

\[ A_{sh} = 2 \times 0.44 = 0.88 \text{ in}^2 \]

\[ N_u = 0.2 V_u = 0.2 \left( 0.85 A_{sh} f_y \right) = 0.2 \left( 0.85 \times 0.88 \times 60 \right) = 8.98 \text{ kips} \]

\[ A_s = 0.6 \text{ in}^2 \]

\[ A_s = \frac{1}{\phi f_y} \left[ V_u \left( \frac{a}{d} \right) + N_u \left( \frac{h}{d} \right) \right] \]

\[ 0.60 = \frac{1}{0.85 \times 60} \left[ V_u \left( \frac{6}{15.06} \right) + 8.98 \left( \frac{15.875}{15.06} \right) \right] \]


\[ V_u = 53.05 \text{ kips} \]

\[ V_n = \frac{53.05}{0.85} = 62.41 \text{ kips} \]

**Direct shear**

\[
A_s = \frac{2V_u}{3f_y \mu_c} + A_n \quad \text{(PCI Eq. 4.6.4)}
\]

\[
A_n = \frac{N_u}{f_y} \quad \text{(PCI Eq. 4.6.5)}
\]

where

\[ \phi = 0.85 \]

\[ f_y = \text{yield strength of } A_s, A_n, A_n, \text{ psi} \]

\[ \mu_c = \frac{1000 \lambda bh \mu}{V_u} \leq 3.4 \text{ (Concrete to Concrete, cast monolithically)} \]

From Fig. 2,

\[ \mu_c = \frac{1000 \lambda bh \mu}{V_u} \leq 3.4, \quad \lambda = 0.75 \text{ for light weight concrete} \]

\[ b = \frac{4.5 + 7}{2} = 5.75 \text{ in.} \]

\[ h = 15\frac{7}{8} = 15.875 \text{ in.} \]

\[ \mu = 1.4 \lambda = 1.4 \times 0.75 = 1.05 \]

\[ N_u = 8.98 \text{ kips} \]

\[ A_s = 0.6 \text{ in}^2 \]

\[ A_s = \frac{2V_u}{3f_y \mu_c} + A_n \]

\[ 0.6 = \frac{2V_u}{3 \times 0.85 \times 60} \times \frac{V_u \times 1000 \text{Psi/ksi}}{1000 \times 0.85 \times 5.75 \times 15.875 \times 1.05} + \frac{8.98}{0.85 \times 60} \]

\[ V_u = 51.40 \text{ kips} \]
\[ V_n = 51.40/0.85 = 60.47 \text{ kips} \]

Check \( \mu_e = \frac{1000 \times 0.85 \times 5.75 \times 15.875 \times 1.05}{51.40 \times 1000} = 1.58 \) (since less than 3.4, OK)

**Diagonal reinforcement at reentrant corner**

\[ A_{sh} = \frac{V_u}{\phi f_y} \quad \text{(PCI Eq. 4.6.7)} \]

where

\[ \phi = 0.85 \]

\( V_u = \) applied factored load

\( A_{sh} = \) vertical or diagonal bars across potential diagonal tension crack, sq in.

\( f_y = \) yield strength of \( A_{sh} \)

From Fig. 2,

\[ A_{sh} = 2 \times 0.44 = 0.88 \text{ in}^2 \]

\( f_y = 60 \text{ ksi} \)

\[ A_{sh} = \frac{V_u}{\phi f_y} \]

\[ 0.88 = \frac{V_u}{0.85 \times 60} \]

\[ V_u = 44.88 \text{ kips} \]

\[ V_n = 44.88/0.85 = 52.8 \text{ kips} \Leftarrow \text{Controls} \]

**Shear reinforcement in the extended end**

\[ \phi V_n = \phi(A_y f_y + A_n f_y + 2 \lambda b d \sqrt{f' c}) \quad \text{(PCI Eq. 4.6.8)} \]

\[ A_n = \frac{N_u}{\phi f_y} \quad \text{(PCI Eq. 4.6.5)} \]
\[ A_b = 0.5(A_s - A_n) \]  
(PCI Eq. 4.6.6)

From Fig. 2,
\[ \phi = 0.85 \]
\[ A_s = 2 \times 0.44 = 0.88 \text{ in}^2 \]

\( A_b \) was not provided in the extended end, \( A_b = 0 \)
\[ f_y = 60 \text{ ksi} \]
\[ \lambda = 0.75, \text{ all light weight concrete} \]
\[ b = 5.75 \text{ in} \]
\[ d = 15.06 \text{ in} \]
\[ f'_c = 6000 \text{ psi} \]
\[ V_u = \phi V_n = \phi(A_s f_y + A_b f_y + 2\lambda bd' \sqrt{f'_c'}) \]
\[ V_n = 0.85 \left( 0.88 \times 60 + 0 + 2 \times 0.75 \times 5.75 \times 15.06 \times \frac{\sqrt{6000}}{1000} \right) = 53.43 \text{ kips} \]
\[ V_n = \frac{53.43}{0.85} = 62.86 \text{ kips} \]

**Anchorage of reinforcement**

According to ACI R12.2

For No. 6 bar
\[ \frac{l_d}{d_b} = \frac{f'_c \alpha \beta \lambda}{25 \sqrt{f'_c'}} \]
\[ \alpha = 1.0, \quad \beta = 1.0, \quad \lambda = 1.3, \quad f_y = 60 \text{ ksi}, \quad f'_c = 6000 \text{ psi}, \quad d_b = \frac{6}{8} \text{ in} \]
\[ l_d = \frac{60000(1)(1.3)(6)}{25 \sqrt{6000}(8)} = 30.21 \text{ in.} \] (OK, 30 in. provided)

For No. 7 bar
\[
\frac{l_d}{d_b} = \frac{f_y \alpha \beta \lambda}{20 \sqrt{f'_c'}}
\]

\[\alpha = 1.0, \beta = 1.3, \lambda = 1.3, f_y = 60 \text{ ksi, } f'_c = 6000 \text{ psi, } d_b = \frac{7}{8} \text{ in.}\]

\[
l_d = \frac{60000(1.3)(1.3)(\frac{7}{8})}{20 \sqrt{6000}} = 57.27 \text{ in. (OK, 60 in. provided)}
\]
List of Tables

Table 1. Concrete Properties
Table 2. Properties of Reinforcing Bars
Table 3. Properties of Mild Steel
Table 4. Summary of Calculations and Test Results
### Table 1. Concrete Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer Strength, $f_{ci}'$ (ksi)</td>
<td>3,095</td>
</tr>
<tr>
<td>Compressive Strength, $f_{cf}'$ (ksi)</td>
<td>6,000</td>
</tr>
<tr>
<td>Modulus of Elasticity at Time of Initial Prestress, $E_{ci}$ (ksi)</td>
<td>2,264</td>
</tr>
<tr>
<td>Modulus of Elasticity, $E_e$ (ksi)</td>
<td>3,152</td>
</tr>
</tbody>
</table>

Note: 1 ksi = 1,000 psi = 6.89 MPa

### Table 2. Properties of Reinforcing Bars

<table>
<thead>
<tr>
<th>Strand Type</th>
<th>Low Relaxation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strand Tensile Strength (ksi)</td>
<td>270</td>
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<tr>
<td>Nominal Diameter (in)</td>
<td>0.5</td>
</tr>
<tr>
<td>Strand Area (in²)</td>
<td>0.153</td>
</tr>
<tr>
<td>$0.7 f_{pu} A_{pu}$ (kip)</td>
<td>28.9</td>
</tr>
<tr>
<td>Modulus of Elasticity, $E_{pu}$ (ksi)</td>
<td>28,322</td>
</tr>
</tbody>
</table>

Note: 1 ksi = 6.89 MPa; 1 kip = 4.45 kN; 1 in = 25.4 mm

### Table 3. Properties of Mild Steel

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity, $E_{mila}$ (ksi)</td>
<td>29,000</td>
</tr>
<tr>
<td>Yield Strength, $f_y$ (ksi)</td>
<td>60</td>
</tr>
<tr>
<td>Permissible Tensile Stress, $f_x$ (ksi)</td>
<td>30</td>
</tr>
</tbody>
</table>

Note: 1 ksi = 6.89 MPa
Table 4. Summary of Calculations and Test Results

<table>
<thead>
<tr>
<th>Code</th>
<th>Shear Span (ft)</th>
<th>Theoretical*</th>
<th>Experimental</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$V_{n(cal)}$ (kip)</td>
<td>$V_{cr}$ (kip)</td>
<td>$V_{test}$ (kip)</td>
</tr>
<tr>
<td>SP I</td>
<td>8</td>
<td>52.80</td>
<td>28.22</td>
<td>45.68</td>
</tr>
<tr>
<td>SP II</td>
<td>5</td>
<td>52.80</td>
<td>29.10</td>
<td>57.40</td>
</tr>
</tbody>
</table>

Note: 1 kip = 4.45 kN; 1 ft = 0.3048m

Legend:

1) $V_c$ = shear resisted by the concrete

$V_{n(cal)}$ = calculated nominal shear capacity at reentrant corner

$V_{cr}$ = shear at first crack at the reentrant corner

$V_{test}$ = maximum shear at failure

2) There is no vertical component of prestress along each stem of tested specimens, therefore $V_p = 0$.

* Calculated shear strength of dapped end based on nominal yield strength of reinforcing bars.
List of Figures

Fig. 1. Lay-Out of Prestressing Strands

Fig. 2. Layout of Mild Reinforcement

Fig. 3. Potential Failure Modes and Required Reinforcement in Dapped-End Connections

Fig. 4. Test Setup of Specimen SP I (8 ft. span)

Fig. 5. Test Setup of Specimen SP II (5 ft. span)

Fig. 6. Failed Side Instrumentation of Specimen SP I (8 ft. span)

Fig. 7. Failed Side Instrumentation of Specimen SP II (5 ft. span)

Fig. 8. Shear-Flexure Failure of Specimen SP I

Fig. 9. Crack Propagation of Specimen SP I

Fig. 10. Shear-Flexure Failure of Specimen SP II

Fig. 11. Crack Propagation of Specimen SP II

Fig. 12. Load vs. Strain at the Reentrant Corners of Specimen SP I

Fig. 13. Load vs. Crack Width at the Reentrant Corners of Specimen SP II

Fig. 14. Load vs. Net Deflection of Specimen SP I

Fig. 15. Load vs. Net Deflection of Specimen SP II
Fig. 1. Lay-Out of Prestressing Strands

Section A-A

1 in. = 2.54 cm

C.G.C. 19.261"

13.875"

12.125"

4\frac{1}{2}" 2"

2"

6"

4"

2"
Fig. 2. Layout of Mild Reinforcement
Fig. 3. Potential Failure Modes and Required Reinforcement in Dapped-End Connections

Notations:

1. Flexure and Axial Tension in the Extended End, which cause the Nib's Flexure Crack

2. Direct Shear Crack

3. Re-entrant Corner Crack

4. Nib Inclined Crack

5. Diagonal Tension Crack

\[ A_s = \text{Area of flexural and axial tension reinforcement} \]

\[ A_{sb} = \text{Area of vertical reinforcement for diagonal cracks} \]

\[ A_v = \text{Area of diagonal tension reinforcement} \]
Fig. 4. Test Setup of Specimen SP I (8 ft. span)

Fig. 5. Test Setup of Specimen SP II (5 ft. span)
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Fig. 8. Shear-Flexure Failure of Specimen SP I

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Fig. 10. Shear-Flexure Failure of Specimen SP II

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Fig. 15. Load vs. Net Deflection of Specimen SP II

1 in. = 25.4mm
1 kip = 4.448kN