Blast Resistance of FRP Retrofitted Un-Reinforced Masonry (URM) Walls With and Without Arching Action

John J. Myers\(^1\), Abdeldjelil Belarbi\(^2\), and Khaled A. El-Domiaty\(^3\)

Modern terrorism is one of the major threats to facilities and personnel throughout the United States and world leading to catastrophic loss in lives and property. The recent escalation in terrorist attacks and its direct impact on the social and economical stability of this nation and other nations worldwide led to an evaluation of the survivability of masonry structures to blast loadings. Un-reinforced masonry (URM) walls have a low resistance against out-of-plane blast loading due to their low flexural capacity and their brittle mode of failure. Failures of URM walls were identified by The Masonry Society (TMS) and the Federal Emergency Management Agency (FEMA) as one of the major causes of material damage and loss of human life due to blast loads. This led to an urgent need in developing effective retrofitting techniques in lieu of impractical conventional approaches to upgrade masonry members to resist blast loads. Several common retrofit techniques have been used for masonry wall systems that involve either (1) adding mass to the wall system by increasing its wall thickness by using a back-up wall system comprised of masonry, concrete or some type of steel framing system or (2) rather than adding mass per say, vertical steel members are added as a back-up system to greatly reduce the span requirements of the masonry wall system. Both of these more conventional approaches have the disadvantage of involving a significant disruption to the occupants in terms of installation time and loss of usable floor space. An alternative method discussed herein is using Fiber Reinforced Polymer (FRP) composites adhered to the surface of the wall to resist high flexural stresses. These systems can be applied to an existing masonry wall system in minimal time with minimal disruption to the occupants and no loss of usable floor space which provides an advantage over other common retrofit techniques. However, the use of FRP is a new approach to blast resistant design and there is little available test data to use as a basis for design of wall upgrades.

The main objective of this research was to demonstrate the performance and implementation of this new technology (application of FRP) for the protection of existing and new masonry buildings against blast loads as an alternative to the traditional concrete or steel retrofit techniques with a view to provide blast resistant design/retrofit guidelines for counteracting terrorist attacks.

As a result, two specific desired outcomes of the research program were (1) to develop an empirical model to predict peak pressure values resulting from an explosion at a given charge weight and standoff distance and (2) to develop an analytical model to predict the dynamic response of the URM walls strengthened with FRP using a simplified single-degree-of-freedom (SDOF) analysis approach. The SDOF model approach is a commonly used method in determining the response of structural components to blast loads.

Masonry walls subjected to blast loads can result in at least three failure modes; tensile failure in zones of high flexure, compressive failure in zones of high flexure, and shear failure near the supports. The walls resist out-of-plane load in the flexural response mode, which creates areas of high moment in the wall. The moment causes tensile strain on the backside face of the wall and compressive strain on the front side facing the blast. Either of these strains can exceed the tensile or compressive failure strain of the reinforced wall and cause flexural failure. Flexural response also causes shear stresses near the supports. Additional shear stresses are caused due to rebound forces and negative blast pressures. These stresses can cause shear failure if they exceed the shear strength of the component and there are no wall to frame connections to transfer the excessive shear force to other structural elements. Shear strength of masonry walls is typically calculated based on the Building Code Requirements for Masonry Structures (1999) and the Uniform Building Code (1994) as shown in Equation 1. Only the webs of the blocks provide the shear strength since all of the walls in this study are un-grouted walls.

\[
V_n = 1.5 \sqrt{f_m' n}
\]

(1) psi Convert to kPa, multiply by 6.89

\(V_n\): masonry shear strength over affective shear area (psi)
\(f_m' n\): masonry unit prism compressive strength (psi)

EXPERIMENTAL PROGRAM

Experimental Design

In order to obtain experimental data concerning blast loading and the associated effects on structures, a pilot
A study was carried out to determine the relationship between the nature of the blast loading (explosion) and the response of the tested masonry wall. Two main parameters were fundamental in determining the blast pressures experienced by a structure. These parameters included the blast charge weight, $Q$, and the standoff distance, $R$. The charge weight is the actual explosive material itself expressed in terms of mass, commonly in units of pounds. The standoff distance is defined as the distance between the explosive charge and the structure under loading, in this case the masonry walls.

Two models were used in this research program to predict peak pressure values and to compare them to the measured pressure values. The first model was based on an empirical equation and reflected pressure coefficients listed in the United States Army technical manual TM5-855-1. The empirical equation, developed by Kingery and Bulmash, computed air blast environment created by the detonation of a hemispherical TNT explosive source at sea level. Information on this model is classified and therefore cannot be introduced in this discussion. The second model is based on an empirical equation presented by the Defense Atomic Support Agency (DASA) Report #1860 (1966). This equation correlates between the peak pressure with the weight and standoff distance as shown below.

$$P_{so} = 150 Q \left( \frac{5}{R} \right)^3$$

where,

- $P_{so}$: Peak pressure at given charge weight and standoff distance in psi
- $Q$: Charge weight of TNT in pounds
- $R$: Radial standoff distance from the center of the explosive to a particular location on a structure, measured in feet.

In addition, the damage that occurred to the masonry walls as a result of the blast loading could naturally be measured and correlated to these two parameters. This correlation would indicate key relationships and provide insight for conducting risk assessment and determining acceptable levels of protection for walls under such blast loadings. As a good assessment tool, four levels of damage were picked to serve as hazard levels to be achieved during the experimental program as recommended by Interim Department of Defense (DOD) Anti-Terrorism/Force Protection Construction Standards. Table 1 illustrates these damage levels for the tested walls, while Table 2 lists the Anti-Terrorism/Force Protection Design Parameters at different threat levels. A minimum threat level with no damage to the structure is required for design purposes. The overall objective of this research was to correlate such data for both URM walls and walls strengthened by means of FRP composites.

**Test Matrix**

A total of eight full-scale walls reflecting different retrofit techniques were included in this study. In these tests, three different types of retrofit techniques were implemented and damage levels to these walls were established as a function of the charge weight and the standoff distance. In this experimental program, the walls were divided into two series. Series I included four walls with nominal dimensions of 88 inches (2.24 m) high by 48 inches (1.22 m) wide by 4 inches (102 mm) thick. In this series, the walls were constructed with two-core hollow concrete blocks that had the nominal size of 4 in. $\times$ 8 in. $\times$ 12 in. (102 mm $\times$ 203 mm $\times$ 305 mm). The net area of a block was 18 in.$^2$ (11,610 mm$^2$) and the net area compressive strength was 1,520 psi (10.34 MPa) calculated from the average of 4 unit tests.

Series II corresponded to the other four walls with nominal dimensions of 88 inches (2.24 m) high by 48 inches (1.22 m) wide by 8 inches (203 mm) thick. The walls in Series II were built with two-core hollow concrete blocks that had the nominal size of 8 in. $\times$ 8 in. $\times$ 16 in. (203 mm $\times$ 203 mm $\times$ 406 mm). The net area of a block was 40 in.$^2$ (25,810 mm$^2$) and the net area compressive strength was 1,810 psi (12.48 MPa) calculated from

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### Table 1. Levels of Damage to Tested URM Walls

<table>
<thead>
<tr>
<th>Level</th>
<th>Damage Level</th>
<th>Damage Description</th>
<th>Performance Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Failure</td>
<td>Wall falls out of test frame.</td>
<td>Wall crumbles and scattered debris.</td>
</tr>
<tr>
<td>II</td>
<td>Heavy Damage</td>
<td>Damage that definitely affects load capacity of wall.</td>
<td>Visible wide-open cracks or significant</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wall will not survive same blast load.</td>
<td>shear cracks, and damage to FRP retrofit.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Small debris close to the wall.</td>
</tr>
<tr>
<td>III</td>
<td>Light Damage</td>
<td>Damage that does not affect load capacity but additional</td>
<td>Hairline to wider cracks at mortar joints or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>damage will be observed under same blast load.</td>
<td>hairline shear cracks.</td>
</tr>
<tr>
<td>IV</td>
<td>No Damage</td>
<td>No damage affecting load capacity of wall.</td>
<td>Hairline cracks in mortar joints.</td>
</tr>
</tbody>
</table>
the average of 4 unit tests. The average compressive strength of the mortar used in both series of tests was 1,500 psi (10.34 MPa). The walls were built by experienced masons using construction techniques representative of good workmanship to not introduce additional variables, such as handwork and different mortar workability that may rise from the construction of the specimens. The specimens were built in two continuous days; half of the wall panel was built during the first day and the other half was built the following day. Then, the specimens were cured under ambient conditions in the vertical position at the test site for at least thirty days before testing. After that, the FRP rods and/or sheets were applied to the walls as recommended by the manufacturer.

Common FRP systems used in civil engineering infrastructure consist of laminates and rods. These systems were used throughout this research study to investigate their effectiveness to resist blast loads. Two applications were investigated including externally bonded laminates where the sheets are attached to the member surface to form a composite laminate; and near surface mounted rods, where the rods are mounted in bed joints between courses near the surface.

GFRP sheets were used in this research study to strengthen the masonry walls. The FRP laminate system consisted of three basic components, namely: putty, impregnating resin and fiber sheets forming the FRP laminate. The manufacturer provided the properties of putty and impregnating resin as illustrated in Table 3. Tensile tests were performed on GFRP laminates to determine their engineering properties. This is illustrated in Table 3. The GFRP sheets were applied to the wall surface by manual lay-up; for their installation a procedure recommended by the manufacturer was followed.

GFRP rods were also used in this research study. This FRP system consisted of two basic components, namely: epoxy-based paste and the rods. The manufacturer provided the properties of the epoxy as illustrated in Table 3. Tensile tests were performed in GFRP rods to determine their engineering properties and is also illustrated in Table 3. The GFRP rods were applied to the wall surface by structural repointing. For their installation, a procedure recommended by the manufacturer was followed.

A summary of the test matrix is shown in Table 4. For Series I, Wall #1 was selected as the un-reinforced control specimen. The remaining three specimens were strengthened with different retrofitting schemes. Thus, Wall #2 was strengthened with 0.25 in. (6.4 mm) GFRP rods at every horizontal joint [i.e. spacing equal to 8 in. (203 mm)]. Wall #3 was strengthened vertically with three 2.5 in. (64 mm) wide GFRP strips [i.e. spacing equal to 9.5 in. (241 mm)], while Wall #4 was strengthened with both 0.25 in. (6.4 mm) GFRP rods at every horizontal joint and three vertical 2.5 in. (64 mm) wide GFRP strips. In this Series, the charges were set up at a standoff distance varying from 6 to 12 ft (1.83 to 3.66 m) reflecting different levels of threat.

Series II was divided into two phases depending on the position of the charge from the walls. In phase one, designated the distant blast phase, the charges were set up at a standoff distance varying from 3 to 12 ft (0.91 to 3.66 m)
reflecting different levels of threat. In phase one, Wall #5 was selected as the un-reinforced control specimen, while Wall #7 was strengthened with 0.25 in. (6.4 mm) GFRP rods at every horizontal joint [i.e. spacing equal to 8 in. (203 mm)]. In phase two, designated the surface blast phase, the blast charges were installed at mid height of the wall. In phase two, Wall #6 was selected as the un-reinforced control specimen, while Wall #8 was strengthened with both 0.25 in. (6.4 mm) GFRP rods at every horizontal joint and two vertical 4 in. (102 mm) wide GFRP strips [i.e. spacing equal to 12 in. (305 mm)]. Figure 1 shows the three types of retrofitting schemes used in this project.

Test Setup

The blast testing of the walls took place at the United States Army Base at Fort Leonard Wood (FLW) near St. Roberts, Missouri. The tests were conducted on a certified military explosives range. The eight walls were constructed on concrete strip footings back-to-back as illustrated in Figure 2. The infill walls had boundary members (concrete beam / footing) on the top and bottom, respectively of the wall. A structural steel frame was designed to withstand the blast loading and support the boundary members and was anchored to the footings. The structural steel frame composed of 6 in. × 6 in. × 3/8 in. (152 mm × 152 mm × 9.5 mm) tube sections and miscellaneous steel plates and angles is shown in Figure 2.

Pressure transducer sensors were used to characterize the pressure wave distribution subjected from the blast loading to the test walls. The pressure transducers were high frequency ICP pressure sensors consisting of 0.218 in. (5.5 mm) diameter probes with a measuring range from 2 to 5,000 psi (14 to 34,000 kPa). Six pressure transducers were inserted into drilled holes on the front face of the wall. Teflon cables of 30 ft (9.14 m) long joined the instrumentation with the data acquisition system (two DAT recorders). The DAT recorders were connected to a portable computer for monitoring and data transfer.

The instrumentation was calibrated at the University of Missouri-Rolla (UMR) Butler-Carlton Civil Engineering Laboratory before conducting the blast tests on the explosive range at FLW. The pressure transducers were subjected to known levels of pressures to later normalize their recorded readings from the field tests. After installation of the instrumentation for test Wall #1 and #2, the sensors were subjected to low levels of blast that did not exceed the elastic range of the weakest unreinforced masonry wall to verify that the sensors were functioning properly and that the values were representative of predicted pressures from the previously mentioned empirical models. This phase was necessary to validate the theoretical pressure values obtained from empirical equations with real experimental values.

After the installation of instrumentation, each wall was ready to be subjected to a series of blast tests. The instrumentation was removed only after the wall system was close to its ultimate capacity to prevent permanent damage to the pressure transducers and accelerometers. The series of blast loads were performed using Pentolite dynamite (50/50 TNT & PETN). The level of blast loads on the walls was selected to reflect different levels of protection as presented previously in Table 2. Since the explosive range where the tests were performed had a limit on the surface charge weight of 5 lbs (2.3 kg), the standoff distance and charge

<table>
<thead>
<tr>
<th>Series Number</th>
<th>Wall Number</th>
<th>Wall Thickness (in.)</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Series I</td>
<td>Wall #1</td>
<td>4</td>
<td>U1</td>
</tr>
<tr>
<td></td>
<td>Wall #2</td>
<td>4</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Wall #3</td>
<td>4</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Wall #4</td>
<td>4</td>
<td>C1</td>
</tr>
<tr>
<td>Series II</td>
<td>Wall #5</td>
<td>8</td>
<td>U2</td>
</tr>
<tr>
<td></td>
<td>Wall #6</td>
<td>8</td>
<td>U3</td>
</tr>
<tr>
<td></td>
<td>Wall #7</td>
<td>8</td>
<td>A2</td>
</tr>
<tr>
<td></td>
<td>Wall #8</td>
<td>8</td>
<td>C2</td>
</tr>
</tbody>
</table>

Legend: U – Unreinforced; A – Retrofit A; B – Retrofit B; C – Retrofit C (1 in. = 25.4 mm)
weight were adjusted to attain these representative threat levels using the aforementioned theoretical pressure models. After each blast event, cracks were monitored and marked until failure, to monitor the progressive damage of the walls. This could later be correlated to the various threat levels investigated. The blast pressure profiles obtained from the pressure transducers was also used to analyze the behavior of the URM walls with FRP as well as in developing a model for predicting the blast peak pressure values at given charge and standoff distance.

**TEST RESULTS AND DISCUSSIONS**

**Empirical Model Development for Peak Pressure Determination**

Pressure wave profiles were obtained from the instrumentation and validated with the predicted pressure values obtained from the two available theoretical models previously mentioned. Figure 3 illustrates a typical pressure wave profile captured during the series of blast tests. It may be noted that a typical pressure wave consists of a sharp compression wave followed by a suction wave. If a wall system can be retrofitted adequately to resist the initial compression pressure wave to a desired threat level, the designer can then allow the wall to fail on the non-occupant side when subjected to the suction wave (rebound forces). Furthermore, the presence of a specific FRP retrofit scheme may then assist in controlling the failure mode and debris level. This approach would naturally require that the boundary system / structural framing system have adequate strength and ductility to withstand the dynamic blast load.

Throughout this research program, different charges of Pentolite dynamite at varying standoff distances ranging from 3 to 20 ft (0.91 to 6.10 m) were used to develop an empirical relationship for pressure distribution based on the measured experimental values obtained. From this data it was observed that there is a linear relationship between charge weight and peak pressure at constant standoff distances as shown in Figure 4. It was also observed that there is an interpolated power function relationship between distance and peak pressures at constant charge weights as shown in Figure 5. These two relationships led to define a general empirical equation as illustrated in Equation 3 to estimate the peak pressure values at a given weight of charge and standoff distance for the Pentolite charge used in this study.

$$P \propto Q R^{-2.5}$$  \hspace{1cm} (3)

(psi) Convert to kPa, multiply by 6.89

where,

- $P$ = Peak pressure value at given charge weight and standoff distance in psi
- $Q$ = Charge weight of Pentolite in pounds
- $R$ = Radial standoff distance from the center of the explosive to a particular location on a structure, measured in feet.

The general form of this relationship does not consider effects due to surrounding structures, topography or other factors that affect the pressure or distribution of the blast wave. Fundamentally this is a general empirical equation

**Figure 3**—Typical Pressure Wave Profile (1 psi = 6.89 kPa)

**Figure 4**—Peak Pressure vs. Charge Weight Relationship at 12 ft (3.66 m) (1 psi = 6.89 kPa; 1 lb= 4.45 N)
for a direct unimpeded blast wave based on the pressure data obtained under this experimental study using Pentolite as the charge material. A correction factor to this expression is required for other charge materials. Pentolite has been reported to have 33% greater mass specific energy compared to TNT, which is often used as the normalized standard for explosive materials [199]. This general empirical equation [Equation (3)] was compared with peak pressure values obtained from the aforementioned theoretical models and found to yield comparable results.

For design purposes, predicted peak pressure values should be greater than the experimental peak pressure values to allow for some level of conservatism. Therefore, using a safety factor of 1.2 is preferred when predicting peak pressure values using Equation 3 as illustrated in Figure 6. Experimental pressure values obtained below 10 psi (69 kPa) carried some noise and vibration values in their waves due to wind or other external factors, and therefore are often inaccurate. However, at higher pressure values the influence of noise and vibration is minimal and the predicted peak pressure values can incorporate a level of conservatism (Figure 6).

Field Observations of Blast Tests

Un-reinforced concrete masonry walls have a high compressive resistance, but have a low tensile resistance. Also, the low rebound forces applied from the plastic behavior of URM walls with the low negative blast pressures are below the shear capacity of the un-reinforced concrete masonry walls. Therefore, the blast capacity of un-reinforced concrete masonry walls was typically limited by their tensile capacity. Then the URM walls only fail out-of-plane in a tensile failure mode in zones of high flexure. This behavior was observed in the un-reinforced walls tested in this research program. It should be noted that the walls tested in this series of tests responded in an impulsive manner due to the close proximity of the charge location where the impulse of the blast wave is dominate. Future work could examine the comparison between the un-reinforced and reinforced walls based on their respective impulse rather than pressure.

Wall U1, for example, experienced vertical and horizontal cracks throughout the blast events until it suffered a brittle tensile failure at its horizontal mid-height mortar joint at blast peak pressure of 182 psi (1254 kPa) that resulted in collapsing of the wall (Figure 7). The FRP practically resisted the tensile stress caused by the flexural response in the retrofitted walls. The masonry cracked at location where the impulse of the blast wave is dominate. Future work could examine the comparison between the un-reinforced and reinforced walls based on their respective impulse rather than pressure.
the bed joints at very low stress level and did not resist the tensile stress beyond cracking stress. However, the concrete masonry resisted much of the compressive stress. Rebound forces resulted from the elastic behavior of the applied FRP to these walls exceeding the shear capacity of the masonry walls at failure. There were no wall-to-frame connections for the retrofitted walls studied in this research program to transfer the rebound forces that resulted. Therefore, the blast capacity of these reinforced concrete masonry walls would typically be limited by their shear capacity expressed by Equation (1). As implied from this equation, the FRP reinforcement does not contribute to the shear strength of the masonry walls. Although the FRP reinforcement along the height and width of the wall may provide some amount of confinement that increases the shear strength of the masonry walls, there is no available data that confirms this statement. Therefore, URM walls retrofitted with FRP are more likely to fail or be heavily damaged in zones of high shear stresses and would not survive additional blast loads without collapsing in an out-of-plane manner.

This behavior was observed in the retrofitted walls of Series I. Wall C1, for example, had light damage levels with hairline shear cracks at both blast peak pressures of 27 and 37 psi (186 and 255 kPa) as shown in Figure 8. It experienced shear damage at blast peak pressure of 182 psi (1,255 kPa) that caused high damage to the wall. The shear damage resulted from high shear stresses and rebound forces caused from the elastic behavior of the GFRP sheets at this blast level. This behavior weakened the wall’s top support to resist higher blast loads. It collapsed in an out-of-plane flexural manner towards the front side of the wall due to the rebound forces caused from the elastic rebound behavior of the GFRP sheets at a blast peak pressure of 274 psi (1,889 kPa) as shown in Figure 9.

The retrofitting schemes changed the behaviors of walls B1 and C1 compared with the un-reinforced masonry control unit wall U1. Wall U1 had no damage at blast pressures of 27 and 37 psi (186 and 255 kPa) and had a brittle sudden flexural failure at blast pressure of 182 psi (1,255 kPa). Wall C1 had an increase in its blast capacity compared to the un-reinforced wall U1, while wall B1’s blast capacity was reduced because of shear limitations causing out-of-plane failure (Figures 10 and 11).

In the standoff blast phase in Series II, Wall A2, retrofitted with 0.25 in. (6.4 mm) diameter GFRP rods horizontally at every bed joint, experienced hairline shear...
cracks along with horizontal and vertical cracks at blast peak pressure of 274 psi (1,889 kPa). This blast pressure caused a light damage level due to the presence of hairline shear cracks. It experienced after that shear damage at blast peak pressure of 460 psi (3,172 kPa) that caused high damage to the wall. The shear damage resulted from high shear stresses and rebound forces at this blast level weakened the wall’s top support to resist higher blast loads. It failed in an out-of-plane flexural manner towards the front side of the wall due to the rebound forces caused from the elastic behavior of the GFRP rods at a blast peak pressure of 1,940 psi (13.4 MPa) as shown in Figure 12b. Comparing behavior of Wall A2 with the un-reinforced masonry control unit Wall U2, the horizontal reinforcement enhanced the flexural capacity of Wall A2 by limiting the rotation and splitting of the vertical cells of the wall in the transverse direction (Figure 12a). Therefore, the overall blast capacity of Wall A2 increased compared to that of the un-reinforced Wall U2.

Wall U2 had a higher out-of-plane flexural resistance than Wall U1 due to its lower slenderness ratio. Wrap around effects at higher blast pressures of the blast wave resulted in a different failure mode of Wall U2 due to its lower slenderness ratio compared to that of Wall U1. The vertical cracks that developed at lower blast levels controlled the behavior of this wall. The corner vertical cells rotated due to the pressure wrap. This behavior resulted in splitting one corner vertical cell of the wall from the rest of it due to tensile failure of the vertical joint at the face of the wall (Figure 13).

From the results and observations obtained from Se-
ries II from the standoff blast phase, it was concluded that 8 in. (203 mm) un-reinforced concrete masonry walls with same slenderness and height to width ratios as in this research program will meet the DOD requirements to resist a minimum threat level with no damage.

In the surface blast phase in Series II, Wall C2, strengthened with both 0.25 in. (6.35 mm) diameter GFRP rods at every horizontal bed joint and three GFRP strips, had a collapse failure at a surface mid-height blast of 5 lbs (2.3 kg) of Pentolite, while the un-reinforced Wall U3 collapsed at a surface mid-height blast of 2 lbs (0.9 kg) of Pentolite. Therefore, the FRP strengthening increased the blast capacity of Wall C2 (Figure 14) with debris scattered directly in front of the wall position. This would help in reducing the hazard of injury and death to building occupants resulting from wall debris scattered at high velocity as was the case of wall U3. Also, the FRP bars and sheets did not rupture or tear from the high impact surface blast loads, so increasing the FRP reinforcement could improve the surface blast capacities and threat levels of these walls.

The instrumentation program enabled the researchers to capture pressure values distributions using calibrated pressure transducers along the height and width of the test walls. The data was then analyzed and curve fit based on charge weight and standoff distance to develop a general empirical equation to estimate the peak pressure values at a given weight of Pentolite charge and standoff distance. This general empirical equation was compared to other theoretical models and found to yield comparable results.

Experimental testing highlighted a shear deficiency in the masonry walls of Series I without arching action. In retrofitting masonry systems for blast, all possible failure modes must be investigated to retrofit for the most desirable failure mechanism. The retrofitting schemes investigated for Series I (non-arching action) indicated that strengthening with FRP laminates appeared to be most desirable since the scatter of debris was less scattered. It also demonstrated a failure mode that was on the non-occupant side (Figure 11). For walls with arching action (Series II), the FRP also appeared to enhance the blast resistance and help control the scatter of debris to a degree. The surface blast tests on the walls with arching action (Series II) also indicated a similar trend. The FRP bonded laminates appeared to provide the best debris scatter control compared to the control walls and walls retrofitted with NSM FRP rods only, particularly for the walls with higher slenderness ratios without arching action.

**ANALYTICAL STUDY**

The basic analytical model used in most blast analysis and design applications is the single-degree-of-freedom (SDOF) system. The use of the SDOF approach to predict the dynamic response of simple structural elements,
Figure 14—Progressive Damage of Wall C2 with Surface Blast Events #1-#3
such as walls, is well documented in a number of sources including *ASCE-Design of Blast Resistant Buildings* (1997) and the Departments of the Army, Navy and Air Force-TM5-1300 (1990) Code. The dynamic resistance is usually specified as a nonlinear function to simulate elastic, perfectly plastic behavior of the structure. The ultimate resistance, *R*ₚ, is reached upon formation of a collapse mechanism in the member. When the resistance is nonlinear, the dynamic equilibrium equation is expressed in Equation (4) where damping is usually conservatively ignored in blast resistant analysis and design due to the short time in which the structure reaches its maximum response.

\[ M \ddot{a} + R = F_t \]

where,

\[ R = \text{lesser of } Ky \text{ or } R_s \]
\[ M = \text{mass} \]
\[ a = \text{acceleration} \]
\[ K = \text{stiffness} \]
\[ y = \text{displacement} \]
\[ F_t = \text{applied force as a function of time} \]

The procedure for obtaining the equivalent SDOF approximation for a structural component is based on its deformed shape under the applied loading and the strain energy equivalence between the actual structure and the SDOF approximation. In addition to strain energy equivalence, the motion of the SDOF system is equivalent to a selected control point on an actual structure. The control point is usually selected at a point of maximum response such as a plastic hinge location within the span. However, the spring force is not equal to the support reactions of the actual member. The equivalent mass, stiffness and loading are obtained throughout by the use of transformation factors to define an equivalent system. Blast design manuals such as TM 5-1300 (Chapter 3) and Biggs (1964) (Chapter 5) contain tabulated transformation factors for typical structural elements. The derivations of the equations for these transformation factors are also given by these references. Transformation factors are used to obtain appropriate properties for the equivalent SDOF system. The dynamic analysis can be performed using these equivalent parameters in place of the corresponding actual values. The alternate form of the bilinear dynamic equilibrium Equation (4) becomes:

\[ M_e \ddot{a} + R_e = F_e \]

Transformation factors also change as the structural member progresses from elastic to plastic ranges. The resistance also changes for the plastic range as shown by Equation (4). In actual practice, an average of the elastic and plastic transformation factors is used in this case.

Response of construction materials under dynamic loads is governed by the stress-strain relationship. Static mechanical properties are readily available from the variety of sources and are well defined by national codes and standard organizations. Specifications referenced in the codes define minimum properties for various grades of material. In practice, the average strength of materials being installed is approximately 25% greater than the specified minimum values. Therefore, a strength increase factor (SIF) is used to account for this condition and is unrelated to strain rate properties of the material. Strength increase factors are used to reduce conservatism and use the full available blast capacity of materials. Blast design manuals such as TM 5-1300 list SIF factors for various materials. A SIF factor of 1.2, the maximum value in the table, is recommended and used in this research program for FRP materials to reduce the conservatism when dealing with these new materials.

Construction materials also experience an increase in strength under rapidly applied loads. These materials cannot respond at the same rate as which the load is applied. At a fast strain rate, a greater load is required to produce the same deformation than at a lower rate. To incorporate the effect of material strength increase with strain, a dynamic increase factor (DIF) is applied to static strength values. DIFs are simply ratios of the dynamic material strength to static strength and are a function of material type as well as strain rate. DIFs are also dependent on the type of stress (i.e. flexural, direct shear) because peak values for these stresses occur at different times. Flexural stresses occur very quickly while peak shears occur relatively late in time resulting in a lower strain rate for shear. Blast design manuals such as TM 5-1300 lists DIF factors for various materials.

When analyzing elements subjected to shock loads using SDOF systems, a time period between the peak deflections as the element vibrates back and forth is needed. This period is called natural period, *tₙ*, and is a function of the element’s mass and stiffness as illustrated in Equation (6).

\[ t_n = 2\pi \sqrt{M_e / K} \]

where,

\[ M_e = \text{equivalent mass} \]
\[ K = \text{stiffness of the SDOF system} \]

Ensuring the adequate response of structural elements to blast loads deformation limits are required. These limits are based on the type of structure or component, construction material used, location of the structure and the desired protection level.

The primary for evaluation of structural response is the evaluation of the ductility ratio and hinge rotations of individual members. Ductility ratio, μ, is defined as the maximum displacement of the member divided by the displacement at the elastic limit. It is a measure of the degree of inelastic response experienced by the member. Empiri-
cal equations and chart solutions have been derived to estimate the demanded ductility ratio as function of blast load and resistance parameters. Empirical equations derivations and references are provided from Biggs (1964) and are illustrated in Equations (7) to (9).

If \( \tau = \left( \frac{t_d}{t_n} \right) < 0.1 \), \( \mu_d = 0.5 \left( \frac{2\pi f I_{so}}{R_u} \right) + 1 \) (7)

If \( \tau = \left( \frac{t_d}{t_n} \right) > 10 \), \( \mu_d = 1 \left( \frac{1-P_{so}/R_u}{2(1-P_{so}/R_u)} \right) \) (8)

In the transition range between these two extreme dynamic responses:

\[
PR_d = \frac{2}{\pi} \left( \frac{2\pi f I_{so}}{R_u} \right) + 1 - \frac{1}{2} \left( \frac{1-P_{so}/R_u}{2(1-P_{so}/R_u)} \right)
\] (9)

where,
- \( t_d \) = blast loading time period
- \( t_n \) = natural period of the SDOF system
- \( \mu_d \) = demand ductility ratio of the member
- \( f \) = natural frequency = \( 1/t_n \)
- \( I_{so} \) = peak blast impulse
- \( P_{so} \) = peak blast pressure
- \( R_u \) = ultimate blast resistance of the SDOF system.

Hinge rotations, \( \theta \), another measure of members response relates the maximum deflection to span and indicates the degree of instability present in critical areas of the members. The angle is formed between a line connecting the endpoints and a line between an endpoint and point of maximum deflection, which is also referred to as support rotation.

Many references such as ASCE Manual 58 and ACI 349 use ductility ratios as the primary gauge of response for concrete and masonry members and treat hinge rotations as secondary criteria for deformation limits. Other references such as TM 5-1300 do not use ductility ratios as deformation limits for concrete and masonry. The relatively stiff nature of concrete and masonry elements produces very high ductility ratios for low maximum deflections. In these cases, ductility ratios may not be indicative of the adequacy of the member and will artificially limit the degree of response. In this manual, hinge rotations are only specified for concrete and masonry elements responding in flexure and are adopted in this research program. Refer to Table 5 for deformation limits for masonry walls.

### Table 5. Deformation Limits for Masonry Walls (TM 5-1300)

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Low Threat Level</th>
<th>Medium Threat Level</th>
<th>High Threat Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-way</td>
<td>0.50</td>
<td>0.75</td>
<td>1.00</td>
</tr>
<tr>
<td>Two-way</td>
<td>0.50</td>
<td>1.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>

**Analysis Study for FRP Retrofitted Masonry Walls**

The response of FRP retrofitted walls was predicted based on the assumption that the walls responded as equivalent single-degree-of-freedom system and that the vertical (longitudinal) FRP reinforcement acted in a manner similar to steel reinforcement, except that the FRP was a brittle reinforcement. For retrofit scheme C, the horizontal FRP reinforcement played a role by increasing the lateral stiffness of the wall and helped in resisting rebound forces produced by the blast wave that were not uniform across the face of the wall; however, analytically the contribution of the horizontal bars was ignored in analytically predicting the blast resistance. Therefore only vertically FRP retrofitted walls were analyzed using SDOF analysis written in a program.

There were two possible failure modes: either failure of masonry in flexure or shear or failure of FRP by rupture. A trial and error approach with several iterations in the program was done to predict the governed failure using equilibrium, compatibility and stress-strain relationships as described below.

\[
\alpha \beta = \frac{e_m}{e_{m'}} - \frac{1}{3} \left( \frac{e_m}{e_{m'}} \right)^2\]

(10)

\[
\alpha \beta \left( 1 - \frac{e_m}{e_{m'}} \right) = \frac{2e_m}{3e_{m'}} - \frac{1}{4} \left( \frac{e_m}{e_{m'}} \right)^2\]

(11)

\[
e_m = \frac{k}{1 - k} e_{f}\] (in the case of masonry failure) (12a)

\[
e_m = \frac{k}{1 - k} e_{ru}\] (in the case of FRP rupture) (12b)

\[
A_f E_f e_f = \alpha \beta k t B f_{w}'
\]

(13)

\[
M_n = \alpha \beta k t \left( \frac{B}{12} \frac{t - B/2}{B/2} \right)
\]

(14)

where,
- \( \alpha \) and \( \beta \) = equivalent stress block factors used for masonry
- \( e_m \) = strain in the masonry
- \( e_{m'} \) = optimum compressive strain in masonry
- \( e_{f} \) = strain in the FRP
εₚ = ultimate strain in FRP
k = ratio factor relating between strain in masonry and FRP
Aₐ = total area of FRP
Eₐ = tensile modulus of FRP
t = thickness of masonry wall
B = width of masonry wall
fₕ′ = compressive strength of masonry
M₀ = moment in the system.

The above equations were used through some iteration with trial values of α, β₁ and k with ultimate masonry and FRP strains in obtaining the actual strains in the masonry and FRP. Then a ratio, kₘ (εₚ / εₚₕ) was calculated. If kₘ was less than 1, then masonry failure governed or vice versa. It was concluded from the analysis, that the failure of masonry always governs the behavior.

After that, the predicted flexural and shear capacities of the reinforced wall were obtained taking into consideration the elastic and plastic behavior of the system and were compared to the required resistances. Also the maximum deflection, support rotation and ductility ratio of the system were calculated to observe the point of failure according to DOD and Army Code TM 5-1300 requirements.

Validation of Analysis Model for FRP Retrofitted Walls

The analytical model was used to analyze the 4 in. (102 mm) wall vertically reinforced on the backside with three 2.5 in. (63.5 mm) wide GFRP strips [i.e. spacing equal to 9.5 in. (241 mm) GFRP sheets (Wall B1)] at different blast loads reflecting the blast events these walls had been through. Table 6 summarizes the single-degree-of-freedom analysis results for Wall B1.

Table 6. SDOF Analysis Results for Wall B1 at Different Blast Events (1 lb = 4.45 N, 1 ft = .031 m, and 1 psi = 6.89 kPa)

<table>
<thead>
<tr>
<th>Charge (lb)</th>
<th>Distance (ft)</th>
<th>Required Resistancea (psi)</th>
<th>Maximum Flexural Resistancea (psi)</th>
<th>Required Shear Resistancea (psi)</th>
<th>Rotation (degrees)</th>
<th>Maximum Deflection (in.)</th>
<th>Ductility Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>16</td>
<td>0.38</td>
<td>1.94</td>
<td>3.27</td>
<td>0.20</td>
<td>0.153</td>
<td>0.60</td>
</tr>
<tr>
<td>0.25</td>
<td>12</td>
<td>0.40</td>
<td>1.93</td>
<td>3.34</td>
<td>0.19</td>
<td>0.145</td>
<td>0.57</td>
</tr>
<tr>
<td>0.50</td>
<td>12</td>
<td>0.52</td>
<td>1.90</td>
<td>3.74</td>
<td>0.23</td>
<td>0.174</td>
<td>0.70</td>
</tr>
<tr>
<td>1.00</td>
<td>12</td>
<td>0.85</td>
<td>1.84</td>
<td>4.44</td>
<td>0.33</td>
<td>0.255</td>
<td>1.05</td>
</tr>
<tr>
<td>1.50</td>
<td>12</td>
<td>1.13</td>
<td>1.90</td>
<td>5.09</td>
<td>0.46</td>
<td>0.355</td>
<td>1.42</td>
</tr>
<tr>
<td>2.00</td>
<td>12</td>
<td>1.38</td>
<td>1.96</td>
<td>5.72</td>
<td>0.61</td>
<td>0.465</td>
<td>1.81</td>
</tr>
<tr>
<td>3.00</td>
<td>12</td>
<td>1.84</td>
<td>2.07</td>
<td>7.00</td>
<td>0.91</td>
<td>0.704</td>
<td>2.40</td>
</tr>
<tr>
<td>1.50</td>
<td>6</td>
<td>2.53</td>
<td>3.15</td>
<td>18.90</td>
<td>1.19</td>
<td>0.900</td>
<td>2.21</td>
</tr>
<tr>
<td>2.00</td>
<td>6</td>
<td>3.20</td>
<td>3.64</td>
<td>24.00</td>
<td>1.53</td>
<td>1.170</td>
<td>2.46</td>
</tr>
</tbody>
</table>

The analytical results shown in Table 6 matched with the actual response of Wall B1, as the wall was governed by its shear capacity when subjected to blast loads. Therefore, increasing the out-of-plane flexural capacity of URM walls has to be associated with proper wall to frame connections with increasing the shear capacity by grouting the wall or reducing span length to resist higher shear stresses and transfer the rebound forces resulted from the retrofit or else the retrofitted wall blast capacity will reduce, as the reinforced wall’s shear capacity controls.

Support rotations computed from the SDOF analysis, a deflection limit for masonry walls adapted by the Department of Defense and the Army Code TM 5-1300 (Refer to Table 5), reflected the actual behavior with the threat and damage levels facing Wall B1 as illustrated in Table 7. The ductility ratio could be also correlated to the damage level facing Wall B1 as illustrated in Table 7. This approach exists in some references and sponsored by some researchers for additional blast design guidelines.

Therefore, it was concluded that the SDOF approach that is commonly used to predict the blast response of structural elements can be used to predict the blast response of URM walls retrofitted with FRP. It was also concluded when comparing of the calculated support rotations and ductility ratios with the test results and the observed damages for the walls have provided some guidelines when retrofitting masonry walls with FRP as illustrated in Table 8. A SDOF analysis for Wall B1, an FRP reinforced masonry wall with laminates, is included as a case study. Based on the support rotation or ductility ratio obtained from a SDOF analysis similar to the example shown, Table 8 can be used as a basic guideline to predict the expected damage level for a given load level.
CONCLUSIONS

This research program has demonstrated that FRP composites offer great benefits for the strengthening of masonry walls to resist blast loads. FRP systems have been proven to increase the out-of-plane flexure capacity of URM elements to resist a higher level of blast threat levels. However, the study has highlighted the associated need to address proper shear capacity requirements and wall to frame connections. The strengthened wall’s shear capacity was the controlling failure mode.

It has also been shown that failure of the FRP strengthened walls without side boundary restrictions failed in a safe manner controlling their debris. Shear damage or failure in a stable manner gave an indication for not surviving additional blast loads. If additional blast loads were applied, the retrofitted walls’ collapsed towards the outside direction in contact debris that would help in reducing the hazard of causing possible harm and injury to building occupants while the un-reinforced walls failed in a sudden flexural manner towards the inside direction with scattered debris.

The test results also showed that the SDOF approach that is commonly used to predict the blast response of structural components can be used to predict the blast response of URM walls strengthened vertically on the backside with FRP laminates.

A comparison of the calculated support rotations and ductility ratios with the test results and the observed damages for the walls and comparing them with the DOD and TM 5-1300 Code requirements (Refer to Tables 1, 2 and 5), has provided some guidelines when retrofitting masonry walls with FRP laminates as shown in the Table 8.

ACKNOWLEDGEMENT

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REFERENCES

ACI (1990), “Code Requirements for Nuclear Safety Related Concrete Structures (ACI349-40) and Commentary (ACI 349R-90),” ACI Committee 349, American Concrete Institute, Farmington Hills, MI, 1990.


NOTATIONS

\[ a \] = acceleration.
\[ A_f \] = total area of FRP.
\[ B \] = width of masonry wall.
\[ E_f \] = tensile modulus of FRP.
\[ f \] = natural frequency = \( 1/ \tau_n \).
\[ f_m^\prime \] = masonry unit prism compressive strength.
\[ F_t \] = applied force as a function of time.
\[ I_{so} \] = peak blast impulse.
\[ k \] = ratio factor relating between strain in masonry and FRP.
\[ K \] = stiffness.
\[ M \] = mass.
\[ M_e \] = equivalent mass.
\[ M_n \] = moment in the system.
\[ P_{so} \] = peak blast pressure.
\[ Q \] = charge weight of TNT in pounds.
\[ R \] = radial standoff distance from the center of the explosive to a particular location on a structure, measured in feet.
\[ R_u \] = ultimate blast resistance.
\[ t \] = thickness of masonry wall.
\[ t_d \] = blast loading time period.
\[ t_n \] = natural period of the SDOF system.
\[ V_u \] = masonry shear strength over affective shear area.
\[ y \] = displacement.
\[ \alpha \text{ and } \beta_f \] = equivalent stress block factors used for masonry.
\[ \varepsilon_m \] = strain in the masonry.
\[ \varepsilon_m^\prime \] = ultimate compressive strain in masonry.
\[ \varepsilon_f \] = strain in the FRP.
\[ \varepsilon_f^\prime \] = ultimate strain in FRP.
\[ \mu_d \] = demand ductility ratio of the member.
SDOF ANALYSIS FOR A RETROFITTED URM WALL WITH FRP LAMINATES (WALL B1)

The response of FRP retrofitted masonry wall B1 was predicted based on the assumption that the walls responded as equivalent single-degree-of-freedom system and that the vertical FRP reinforcement acted in a manner similar to steel reinforcement, except that the FRP was a brittle reinforcement. It was assumed two possible failure modes: failure of masonry or rupture of FRP. Wall B1 had the same properties and dimensions of Wall U1, and this wall was also evaluated at the same blast threat. The analysis is compared to a 1.0 lb (4.45 N) charge weight at 12 ft 0 in. (305 mm) stand off distance.

Properties of GFRP Sheets:
Area per sheet = 0.12 in² (0.774 cm²) and 
Number of sheets = 3
Therefore, total fiber area,
\[ A_f = (0.12)(3) = 0.36 \text{ in}^2 (2.32 \text{ cm}^2) \]
Center to center spacing,
\[ S = 12 \text{ in.} (30.48 \text{ cm}) \]
GFRP tensile strength,
\[ f'_u = 240 \text{ ksi} (1654.74 \text{ MPa}) \]
GFRP tensile modulus,
\[ E_f = 12,000 \text{ ksi} (82,737.09 \text{ MPa}) \]
For dynamic flexure, *(Refer to TM 5-1300)*
\[ f'_u = (SIF)(DIF) \]
\[ f'_u = (1.3)(1.05)(240) = 327.6 \text{ ksi} (2258.72 \text{ MPa}) \]
Therefore, \[ \varepsilon'_m = f'_u/E_f = 327.6/12,000 = 0.0273 \](ultimate strain in GFRP sheet)

Two possible failures: *masonry failure* or *FRP rupture*. A trial and error approach with several iterations was done to predict the governed failure using equilibrium, compatibility and stress-strain relationships as shown below.

**Masonry Failure:**
Assume \( \varepsilon'_m = 0.002 \)
(optimum masonry compressive strain)
Assume \( \varepsilon'_m = 0.0035 \)
(ultimate masonry compressive strain)
Guess values for \( \alpha \) and \( \beta_1 \)
Let \( \alpha = 1 \) and \( \beta_1 = 1 \)
Assume the current strain in masonry is at ultimate,
Therefore, \( \varepsilon'_m = \varepsilon''_m = 0.0035 \)
The following relations were used to estimate the actual \( \alpha \) and \( \beta \) values:
\[
\alpha \beta \beta_1 \varepsilon''_m = \varepsilon'_m - \varepsilon'_m \left( \frac{1}{3} \varepsilon'_m \right)^2
\]
*(Refer to Equation 10)*
\[
\alpha \beta_1 = \left( 1 - \frac{\beta_1}{2} \right) = \frac{2 \varepsilon'_m}{3 \varepsilon''_m} - \frac{1}{4} \left( \frac{\varepsilon'_m}{\varepsilon''_m} \right)^2
\]
*(Refer to Equation 11)*
Therefore, \( \alpha = 0.81 \) and \( \beta_1 = 0.9 \)
Assume, \( k = 0.5 \) and \( \varepsilon_f = \varepsilon''_m = 0.0273 \)
These other two relations were used to obtain the actual values of \( k \) and \( \varepsilon_f \)
\[
\varepsilon''_m = \frac{k}{1-k} \varepsilon_f
\]
*(Refer to Equation 12a)*
\[
A_v E_f \varepsilon_f = A_f E_f B \frac{1}{12} \left( t - \frac{\beta_1}{2} kt \right)
\]
*(Refer to Equation 13)*
Therefore, \( k = 0.227 \) and \( \varepsilon_f = 0.012 \)
Calculating the moment in the system:
\[
M_n = \alpha \beta \beta_1 \left( f'_u \frac{B}{12} \left( t - \frac{\beta_1}{2} kt \right) \right)
\]
*(Refer to Equation 14)*
\[
M_n = (0.81)(0.9)(0.227)(3.625)(1.785)
\]
\[
\frac{48}{12} \left( 3.625 - \frac{0.9}{2} (0.227)(3.625) \right)
\]
\[
M_n = 13.952 \text{ kft} (18.97 \text{ kN} \cdot \text{m})
\]
\[
k_m = \varepsilon_f/\varepsilon''_m = 0.012/0.0273 = 0.44
\]
Since \( k_m < 1 \), therefore masonry controlled, but still check fiber rupture.

**Fiber Rupture:**

\[
\alpha \beta = \frac{e_m - 1}{3} \left( \frac{e_m}{e_m} \right)^2 \tag{A-1}
\]

(Refer to Equation 10)

\[
\alpha \beta = \left( 1 - \frac{\beta}{2} \right) = \frac{2e_m - 1}{4} \left( \frac{e_m}{e_m} \right)^2 \tag{A-2}
\]

(Refer to Equation 11)

Therefore, \( e_m = k \frac{\varepsilon_{fu}}{1-k} \) (Refer to Equation 12a)

\[
A_f E_f \varepsilon_f = \alpha \beta \frac{kt_b f_{ad}}{L} \tag{A-4}
\]

No solution found for \( k, e_m, \alpha \) or \( \beta \)

Therefore, \( k_m \) still equal to 0.44 and therefore masonry controlled.

Computing Required Shear Resistance: (based on unit inch width of the wall)

Maximum shear force (reaction) in system (Refer to TM 5-1300 Transformation Factor Table)

\[
V_o = 0.38R + 0.12P_{L} \pm M/L \tag{A-10}
\]

\[
= 0.38(474.83) + 0.12(16.13)(88) \pm(13.95)(1000)(12)/(88)(48)
\]

Therefore, maximum support dynamic reaction, \( V_o = 390.4 \) lb (1736.6 N)

Therefore, required shear resistance, \( V_{uo} = 390.4/(88)(1) = 4.44 \) psi (30.61 kPa)

Since \( V_{uo} < V_o \), therefore this retrofitted wall has no problem with shear at this blast threat.

Moment at mid-span:

\[
M_{wp} = V_o (L/2) - P_{sL} (L^2/8) \tag{A-11}
\]

\[
= [390.4(88/2)-16.13(88^2/8)] \pm(13.95)(1000)(12)/(88)(48)
\]

Therefore, elastic deflection, \( y_e = \frac{R_u}{K_1} \) (Refer to TM 5-1300 Transformation Factor Table)

\[
K_1 = 185E_m I_e /L^3 \tag{A-12}
\]

Elastic range:

\[
K_{el} = 0.45, K_{el} = 0.58, K_{el} = 0.45/0.58 = 0.776 \tag{A-21}
\]
Elastic-Plastic range:
\[ K_{M2} = 0.5, \quad K_{L2} = 0.64 \]  \hspace{1cm} (A-22)
\[ K_{LM2} = K_{M2}/K_{L2} = 0.5/0.64 = 0.781 \]

Plastic range:
\[ K_{M3} = 0.33, \quad K_{L3} = 0.5 \]  \hspace{1cm} (A-23)
\[ K_{LM3} = K_{M3}/K_{L3} = 0.33/0.5 = 0.66 \]
\[ K_{LM} = (K_{LM1} + K_{LM2} + K_{LM3})/3 = 0.739 \]

Equivalent mass (per unit width of wall),
\[ M = 597.82 \text{ psi-ms}^2/\text{in.} \quad (1,622.7 \text{ kPa-ms}^2/\text{cm}) \]  \hspace{1cm} (A-24)
\[ M_e = (K_{LM})(M) = (0.739)(597.82) = 441.8 \text{ psi-ms}^2/\text{in.} \quad (1,199.2 \text{ kPa-ms}^2/\text{cm}) \]

Natural Period,
\[ t_n = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{441.8/7.63} = 47.81 \text{ ms} \]  \hspace{1cm} (A-25)

Duration-Period ratio,
\[ \tau = t/t_n = 1.83/47.81 = 0.038 \]  \hspace{1cm} (A-26)

Since, \( \tau = 0.038 < 0.1 \), Therefore use Equation 7.

Therefore,
\[ \mu_d = 0.5\{(2\pi f / R)^2 + 1\} \]
\[ = 0.5\{(2\pi)(1/47.81)(14.77)/1.84)^2 + 1\} \]
\[ = 1.056 \]  \hspace{1cm} (A-27)

Maximum deflection,
\[ y_m = (\mu_d)(y_e) = (1.056)(0.24) \]
\[ = 0.255 \text{ in} \quad (0.65 \text{ cm}) \]  \hspace{1cm} (A-28)

Support rotation,
\[ \theta = \tan^{-1}[y_m/(0.5L)] \]
\[ = 0.332 \text{ degree} < 1 \text{ degree} \quad \text{ok} \]  \hspace{1cm} (A-29)

According to the Deformation limit requirements reported by *TM 5-1300* (Table 5), the wall is in a low threat level (below 0.5 degrees) and adequate to resist the specified blast load. Therefore, the vertical GFRP sheet reinforcement increased the blast capacity of the wall to resist the specified load in a low threat level.